DIFFERENTIATION ROADMAP

Which rule do I use? Follow the steps from top to bottom.

Math with Alex Academy
IB Math AA HL · Calculus
Lessons 5.3 – 5.10

STEP 1 Rewrite it first

Simplify before you differentiate. ▶ 5.4

Index form $\sqrt{x} = x^{\frac{1}{2}}, \quad \frac{1}{x^3} = x^{-3}$

Log laws $\ln \frac{A}{B} = \ln A - \ln B, \quad \ln A^n = n \ln A$

Term by term $(2f + g)' = 2f' + g'$, constants wait in front

STEP 2 One box: use its rule

The box method is the chain rule, written faster. Click a ▶ lesson number to watch that lesson.

□ = box (the inside function), □′ = box prime (the derivative of the box). Every rule works the same way: **the outside changes**, **the box stays exactly the same**, then **multiply by box prime**. Never skip box prime.

Power rule ▶ 5.5

$$(\square^{\,n})' = n\,\square^{\,n-1} \cdot \square'$$

box is x: $(x^n)' = nx^{n-1}$, because $\square' = 1$

Square root ▶ 5.5

$$(\sqrt{\square})' = \frac{\square'}{2\sqrt{\square}}$$

the power rule with n = ½, ready to use

Exponential ▶ 5.7

$$(e^{\square})' = e^{\square} \cdot \square' \cdot \ln e$$

other base: $(a^{\square})' = a^{\square} \cdot \square' \cdot \ln a$

Logarithm ▶ 5.7

$$(\ln \square)' = \frac{\square'}{\square \cdot \ln e}$$

other base: $(\log_a \square)' = \frac{\square'}{\square \cdot \ln a}$

sin ▶ 5.8

$$(\sin \square)' = \cos \square \cdot \square'$$

outside changes, same box inside

cos ▶ 5.8

$$(\cos \square)' = -\sin \square \cdot \square'$$

co-function: minus sign

tan ▶ 5.8

$$(\tan \square)' = \sec^2 \square \cdot \square'$$

outside changes, same box inside

sec ▶ 5.8

$$(\sec \square)' = \sec \square \tan \square \cdot \square'$$

outside changes, same box inside

cot ▶ 5.8

$$(\cot \square)' = -\operatorname{cosec}^2 \square \cdot \square'$$

buddy swap from tan:
$\tan \to -\cot, \ \sec \to \operatorname{cosec}$

cosec ▶ 5.8

$$(\operatorname{cosec} \square)' = -\operatorname{cosec} \square \cot \square \cdot \square'$$

buddy swap from sec

arcsin ▶ 5.9

$$(\arcsin \square)' = \frac{\square'}{\sqrt{1 - \square^2}}$$

$(\arccos \square)'$: the same with a minus sign

arctan ▶ 5.9

$$(\arctan \square)' = \frac{\square'}{1 + \square^2}$$

no square root

The ln of the base is always there
Exponentials: multiply by $\ln(\text{base})$. Logs: divide by $\ln(\text{base})$. With base e it is hidden, because $\ln e = 1$.

Learn four trig rules
sin, cos, tan and sec. The buddy swap gives cot and cosec.

Watch the minus signs
cos, cot, cosec and arccos all have a minus sign in their derivatives.

more than one part?

STEP 3 Look at the structure

Product, quotient, layers, or x and y mixed together?

PRODUCT ▶ Lesson 5.6

Product rule

When: two parts multiplied, $y = uv$

$$y' = u'v + uv'$$

- Differentiate the first, keep the second. Then keep the first, differentiate the second
- Each part may still need the box method
- Take out the common factor at the end

QUOTIENT ▶ Lesson 5.6

Quotient rule

When: one part over another, $y = \frac{u}{v}$

$$y' = \frac{u'v - uv'}{v^2}$$

- The order matters: top first. The wrong order flips the sign
- Keep the bracket after the minus
- Only a constant on top? Index form and the power rule are faster

LAYERS ▶ 5.5 ▶ 5.7 ▶ 5.8

Boxes inside boxes

When: a function inside a function inside a function

$$\left[\log_5(e^x - 3)\right]^3$$

- Find the outer box first and use its rule
- Work your way in, one layer at a time
- Every layer gives one piece of the answer, and its own box prime

IMPLICIT ▶ Lesson 5.10

Implicit differentiation

When: x and y are mixed, like $x^3 + y^3 = 9xy$

- Differentiate every term and write every box prime
- $\frac{dx}{dx} = 1$ disappears, $\frac{dy}{dx}$ stays: $(y^2)' = 2y\frac{dy}{dx}$
- Collect the $\frac{dy}{dx}$ terms, factorise, divide
- A variable to a variable power, like x^x: take ln of both sides first

From first principles (Lesson 5.3) ▶ 5.3

Use the definition only when the question says “from first principles”:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Every time

- Ask first: one box, a product or a quotient? The full flowchart is on page 2
- The box must stay the same until the very last step
- Tidy the answer: positive powers, common factors, one fraction

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WHICH RULE?

Differentiation: ask the questions in order, from top to bottom.

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START $\dfrac{dy}{dx} = ?$ — $\square$ = box, the inside function · $\square'$ = box prime, its derivative (Lesson 5.5) · ▶ 5.5 = click to watch that lesson

QUESTION 1
Can it be written more simply?
A root, a fraction with one term on the bottom, or the ln of a product, quotient or power.

YES →

Rewrite first — ▶ 5.4
index form or the log laws, then carry on down

$$y = \frac{x^2 - 5}{x} = x - 5x^{-1} \;\Rightarrow\; \frac{dy}{dx} = 1 + 5x^{-2}$$

$$y = \ln\frac{x-1}{x+1} = \ln(x-1) - \ln(x+1) \;\Rightarrow\; \frac{dy}{dx} = \frac{1}{x-1} - \frac{1}{x+1}$$

NO ▼

QUESTION 2
Are x and y mixed together?
y is not the subject of the equation.

YES →

Implicit differentiation — ▶ 5.10
differentiate every term: $\frac{dx}{dx} = 1$ disappears, $\frac{dy}{dx}$ stays

$$x^2 + y^2 = 16 \;\Rightarrow\; 2x + 2y\frac{dy}{dx} = 0 \;\Rightarrow\; \frac{dy}{dx} = -\frac{x}{y}$$

NO ▼

QUESTION 3
Is it two parts multiplied?

YES →

Product rule — ▶ 5.6
$y = uv \;\Rightarrow\; y' = u'v + uv'$

$$y = x^2(x+1)^5 \;\Rightarrow\; \frac{dy}{dx} = 2x(x+1)^5 + x^2 \cdot 5(x+1)^4 = x(x+1)^4(7x+2)$$

NO ▼

QUESTION 4
Is it one part over another?

YES →

Quotient rule — ▶ 5.6
$$y = \frac{u}{v} \;\Rightarrow\; y' = \frac{u'v - uv'}{v^2}$$

$$y = \frac{x^2+1}{x+3} \;\Rightarrow\; \frac{dy}{dx} = \frac{2x(x+3) - (x^2+1)\cdot 1}{(x+3)^2} = \frac{x^2+6x-1}{(x+3)^2}$$

NO ▼

QUESTION 5
It is one box. What is on the outside?

Find the box, keep it exactly the same, change the outside with its rule, then multiply by box prime.

▼ **a power or a root** — $\square^n$, $\sqrt{\square}$

Power rule — ▶ 5.5
$$y = (2x^3 - 7)^4$$
$$\frac{dy}{dx} = 4(2x^3-7)^3 \cdot 6x^2$$
$$= 24x^2(2x^3-7)^3$$

▼ **an exponential** — $e^{\square}$, $a^{\square}$

× ln(base) — ▶ 5.7
$$y = e^{x^2}: \quad e^{x^2} \cdot 2x \cdot \ln e$$
$$y = 5^{3x}: \quad 5^{3x} \cdot 3 \cdot \ln 5$$

▼ **a logarithm** — $\ln\square$, $\log_a\square$

÷ ln(base) — ▶ 5.7
$$y = \ln(x^2+1): \quad \frac{2x}{(x^2+1)\cdot \ln e}$$
$$y = \log_2(3x+1): \quad \frac{3}{(3x+1)\cdot \ln 2}$$

▼ **trig or inverse trig** — $\sin\square$, $\arctan\square$, …

Trig templates — ▶ 5.8
$$y = \cos(x^2): \quad -\sin(x^2)\cdot 2x$$
$$y = \arctan(x^2): \quad \frac{2x}{1+x^4}$$

Three more cases

Boxes inside boxes — ▶ 5.7
peel the layers from the outside in: power, then log, then e

$$y = \left[\log_5(e^x - 3)\right]^3$$
$$\frac{dy}{dx} = \frac{3\left[\log_5(e^x-3)\right]^2 \cdot e^x}{(e^x-3)\cdot \ln 5}$$

A variable to a variable power — ▶ 5.10
take ln of both sides, then differentiate implicitly

$$y = x^x \;\Rightarrow\; \ln y = x\ln x$$
$$\frac{1}{y}\frac{dy}{dx} = \ln x + 1 \;\Rightarrow\; \frac{dy}{dx} = x^x(\ln x + 1)$$

"From first principles" — ▶ 5.3
only when the question asks for it: use the definition

$$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$

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